sandwich thm

$$g(x) \le f(x) \le h(x)$$
$$\searrow \quad \downarrow \quad \swarrow$$
$$L$$

Feb 1-10:09 AM

8.2 l'hopital's rule

$$\lim_{x\to 2} \frac{\sin(x-2)}{(x-2)^2+\frac{1}{2}(x-2)} \overset{?}{=} 2 \quad \cancel{=}\ ? \quad \frac{0}{0}$$

undefined

Indeterminate form

replace functions with their slopes (der)

$$\lim_{x\to 2} \frac{\sin(x-2)}{(x-2)^2+\frac{1}{2}(x-2)} = \lim_{x\to 2} \frac{\cos(x-2)}{2(x-2)+\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

Feb 1-10:17 AM

l'hopital's rule

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

also ok for $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

must be indet.

$\frac{0}{0}$ $f(a)=0,\ g(a)=0$

or $\frac{\infty}{\infty}$ $f(a)\to\infty,\ g(a)\to\infty$

Feb 1-10:34 AM

$$\lim_{x\to 0} \frac{\sin x}{x^2} = \lim_{x\to 0} \frac{\cos x \nearrow 1}{2x \searrow 0} = \infty$$

$\frac{0}{0}$

$$\lim_{x\to \frac{\pi}{2}} \frac{\sec x}{1+\tan x} = \lim_{x\to \frac{\pi}{2}} \frac{\cancel{\sec} x \tan x}{\sec^{\cancel{2}} x}$$

$\frac{\infty}{\infty}$ $= \lim_{x\to \frac{\pi}{2}} \frac{\sin x}{\cancel{\cos x}} \frac{\cancel{\cos x}}{1} = 1$

Feb 1-10:40 AM

other indet. forms: $\infty \cdot 0$, $\infty - \infty$, 1^∞, 0^0, ∞^0

can't use l'hopital's until we transform to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \underbrace{x}_{\infty} \cdot \underbrace{\sin\frac{1}{x}}_{0} = 1$$

$$\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0} \quad = \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = 1$$

Feb 1-10:48 AM

$\infty - \infty$

$$\lim_{x\to 1} \frac{(x-1)\,1}{(x-1)\ln x} - \frac{1}{x-1}\,\frac{\ln x}{\ln x}$$

$$\lim_{x\to 1} \frac{x-1-\ln x}{(x-1)\ln x} \quad \frac{0}{0}$$

$$\lim_{x\to 1} \frac{\left(1-\frac{1}{x}\right)x}{\left((x-1)\frac{1}{x}+\ln x\cdot 1\right)x} = \lim_{x\to 1}\frac{x-1}{x-1+x\ln x}$$

$$= \lim_{x\to 1}\frac{1}{1+x\cdot\frac{1}{x}+\ln x\cdot 1} = \frac{1}{2}$$

Feb 1-10:55 AM

read Ex ~~#~~ 8, 9, 10

Feb 1-11:01 AM